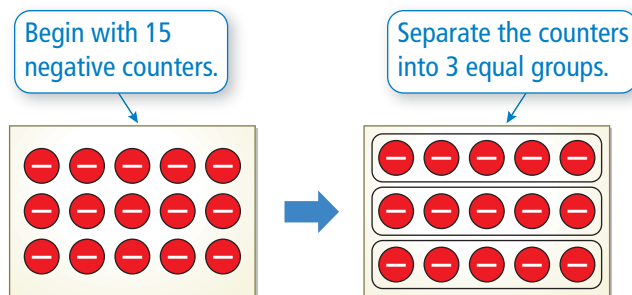


1.5 Dividing Integers

Essential Question Is the quotient of two integers *positive*, *negative*, or *zero*? How can you tell?

1 EXAMPLE: Dividing Integers with Different Signs

Use integer counters to find $-15 \div 3$.



Because there are five negative counters in each group, $-15 \div 3 = -5$.

2 ACTIVITY: Rewriting a Product as a Quotient

Work with a partner. Rewrite the product $3 \cdot 4 = 12$ as a quotient in two different ways.

First Way

12 is equal to 3 groups of .

So, $12 \div 3 = \text{}$.

Second Way

12 is equal to 4 groups of .

So, $12 \div 4 = \text{}$.

3 EXAMPLE: Dividing Integers with Different Signs

Rewrite the product $-3 \cdot (-4) = 12$ as a quotient in two different ways. What can you conclude?

First Way

$$12 \div (-3) = -4$$

Second Way

$$12 \div (-4) = -3$$

In each case, when you divide a positive integer by a negative integer, you get a negative integer.

4 EXAMPLE: Dividing Negative Integers

Rewrite the product $3 \cdot (-4) = -12$ as a quotient in two different ways. What can you conclude?

First Way

$$-12 \div (-4) = 3$$

Second Way

$$-12 \div (3) = -4$$

❖ When you divide a negative integer by a negative integer, you get a positive integer. When you divide a negative integer by a positive integer, you get a negative integer.

Inductive Reasoning

Work with a partner. Complete the table.

	Exercise	Type of Quotient	Quotient	Quotient: Positive, Negative, or Zero
1	5. $-15 \div 3$	Integers with different signs		
2	6. $12 \div 4$			Positive
3	7. $12 \div (-3)$		-4	
4	8. $-12 \div (-4)$	Integers with the same sign		Positive
	9. $-6 \div 2$			
	10. $-21 \div (-7)$			
	11. $10 \div (-2)$			
	12. $12 \div (-6)$			
	13. $0 \div (-15)$			
	14. $0 \div 4$			

What Is Your Answer?

- IN YOUR OWN WORDS** Is the quotient of two integers *positive*, *negative*, or *zero*? How can you tell?
- Write general rules for dividing (a) two integers with the same sign and (b) two integers with different signs.

Practice

Use what you learned about dividing integers to complete Exercises 8–15 on page 32.

Key Ideas

Dividing Integers with the Same Sign

Words The quotient of two integers with the same sign is positive.

Numbers $8 \div 2 = 4$ $-8 \div (-2) = 4$

Dividing Integers with Different Signs

Words The quotient of two integers with different signs is negative.

Numbers $8 \div (-2) = -4$ $-8 \div 2 = -4$

EXAMPLE 1 Dividing Integers with the Same Sign

Find $-18 \div (-6)$.

The integers have the same sign.

$$-18 \div (-6) = 3$$

The quotient is positive.

∴ The quotient is 3.

EXAMPLE 2 Dividing Integers with Different Signs

Divide.

a. $75 \div (-25)$

b. $\frac{-54}{6}$

The integers have different signs.

$$75 \div (-25) = -3$$

$$\frac{-54}{6} = -9$$

The quotient is negative.

∴ The quotient is -3 .

∴ The quotient is -9 .

On Your Own

Divide.

1. $14 \div 2$

2. $-32 \div (-4)$

3. $-40 \div (-8)$

4. $0 \div (-6)$

5. $\frac{-49}{7}$

6. $\frac{21}{-3}$

Now You're Ready
Exercises 8–23

EXAMPLE 3 Evaluating Expressions

Remember

Use order of operations when evaluating an expression.



Evaluate $10 - x^2 \div y$ when $x = 8$ and $y = -4$.

$$\begin{aligned} 10 - x^2 \div y &= 10 - 8^2 \div (-4) && \text{Substitute 8 for } x \text{ and } -4 \text{ for } y. \\ &= 10 - 8 \cdot 8 \div (-4) && \text{Write } 8^2 \text{ as repeated multiplication.} \\ &= 10 - 64 \div (-4) && \text{Multiply 8 and 8.} \\ &= 10 - (-16) && \text{Divide 64 and } -4. \\ &= 26 && \text{Subtract.} \end{aligned}$$

On Your Own

Now You're Ready
Exercises 28–31

Evaluate the expression when $a = -18$ and $b = -6$.

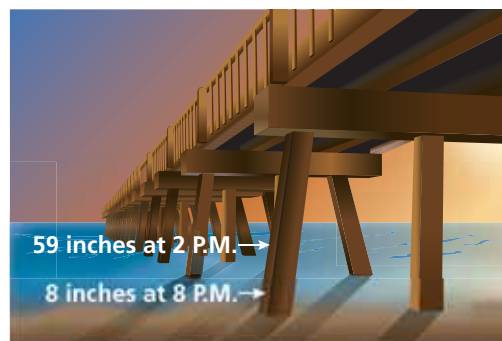
7. $a \div b$

8. $\frac{a+6}{3}$

9. $\frac{b^2}{a} + 4$

EXAMPLE 4 Real-Life Application

You measure the height of the tide using support beams of a pier. Your measurements are shown in the picture. What is the mean hourly change in the height?



Use a model to solve the problem.

$$\begin{aligned} \text{Mean hourly change} &= \frac{\text{Final height} - \text{Initial height}}{\text{Elapsed Time}} \\ &= \frac{8 - 59}{6} && \text{Substitute. The elapsed time from 2 P.M. to 8 P.M. is 6 hours.} \\ &= \frac{-51}{6} && \text{Subtract.} \\ &= -8.5 && \text{Divide.} \end{aligned}$$

∴ The mean change in the height of the tide is -8.5 inches per hour.

On Your Own

10. The height of the tide at the Bay of Fundy in New Brunswick decreases 36 feet in 6 hours. What is the mean hourly change in the height?

Vocabulary and Concept Check

- WRITING** What can you tell about two integers when their quotient is positive? negative? zero?
- VOCABULARY** A quotient is undefined. What does this mean?
- OPEN-ENDED** Write two integers whose quotient is negative.
- WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$$\frac{10}{-5}$$

$$\frac{-10}{5}$$

$$\frac{-10}{-5}$$

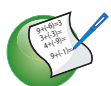
$$-\left(\frac{10}{5}\right)$$

Tell whether the quotient is *positive* or *negative* without dividing.

5. $-12 \div 4$

6. $\frac{-6}{-2}$

7. $15 \div (-3)$



Practice and Problem Solving

Divide, if possible.

- | | | | | | |
|---|---|----------------------|---------------------|----------------------|----------------------|
| 1 | 2 | 8. $4 \div (-2)$ | 9. $21 \div (-7)$ | 10. $-20 \div 4$ | 11. $-18 \div (-6)$ |
| | | 12. $\frac{-14}{7}$ | 13. $\frac{0}{6}$ | 14. $\frac{-15}{-5}$ | 15. $\frac{54}{-9}$ |
| | | 16. $-33 \div 11$ | 17. $-49 \div (-7)$ | 18. $0 \div (-2)$ | 19. $60 \div (-6)$ |
| | | 20. $\frac{-56}{14}$ | 21. $\frac{18}{0}$ | 22. $\frac{65}{-5}$ | 23. $\frac{-84}{-7}$ |

ERROR ANALYSIS Describe and correct the error in finding the quotient.

24.

$$\frac{-63}{-9} = -7$$

25.

$$0 \div (-5) = -5$$

- ALLIGATORS** An alligator population in a nature preserve in the Everglades decreases by 60 alligators over 5 years. What is the mean yearly change in the alligator population?
- READING** You read 105 pages of a novel over 7 days. What is the mean number of pages you read each day?

3 **ALGEBRA** Evaluate the expression when $x = 10$, $y = -2$, and $z = -5$.

28. $x \div y$

29. $\frac{10y^2}{z}$

30. $\left| \frac{xz}{-y} \right|$

31. $\frac{-x^2 + 6z}{y}$

Find the mean of the integers.

32. $3, -10, -2, 13, 11$

33. $-26, 39, -10, -16, 12, 31$

Evaluate the expression.

34. $-8 - 14 \div 2 + 5$

35. $24 \div (-4) + (-2) \cdot (-5)$

36. **PATTERN** Find the next two numbers in the pattern $-128, 64, -32, 16, \dots$. Explain your reasoning.

37. **HIKING** While hiking along the Croom Tract Loop section of the Florida Trail, a hiker climbs down an 84-foot hill in 4 minutes. What is the mean change in elevation per minute?

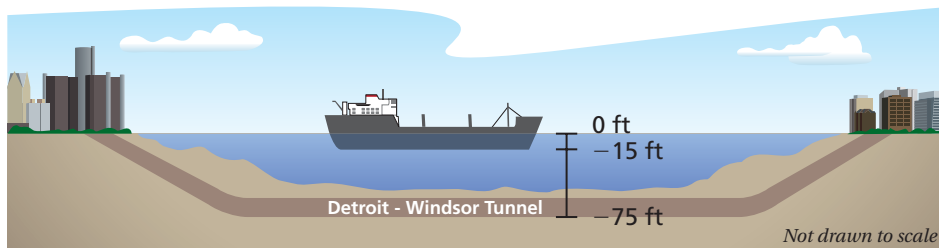
38. **THE MASTERS** In 1997, at the age of 21, Tiger Woods became the youngest golfer to win the Masters Tournament. The table shows his score for each round.

Scorecard	
Round 1	-2
Round 2	-6
Round 3	-7
Round 4	-3

a. Tiger set the tournament record with the lowest total score. What was his total score?

b. What was his mean score per round?

39. **TUNNEL** The Detroit-Windsor Tunnel is an underwater highway that connects the cities of Detroit, Michigan, and Windsor, Ontario. How many times deeper is the roadway than the bottom of the ship?



40. **AMUSEMENT PARK** The regular admission price for an amusement park is \$72. For a group of 15 or more, the admission price is reduced by \$25. How many people need to be in a group to save \$500?

41. **Number Sense** Write five different integers that have a mean of -10 . Explain how you found your answer.



Fair Game Review What you learned in previous grades & lessons

Graph the values on a number line. Then order the values from least to greatest.

42. $-6, 4, |2|, -1, |-10|$

43. $3, |0|, |-4|, -3, -8$

44. $|5|, -2, -5, |-2|, -7$

45. **MULTIPLE CHOICE** What is the value of $4 \cdot 3 + (12 \div 2)^2$?

0

(A) 15

(B) 48

(C) 156

(D) 324